Digital Circuits

ECS 371

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Lecture 9

Office Hours:
BKD 3601-7
Monday 9:00-10:30, 1:30-3:30
Tuesday 10:30-11:30

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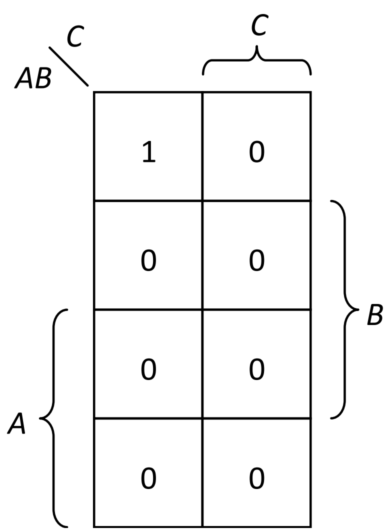
Announcement

- HW3 posted on the course web site
  - Chapter 4: 5(b,d), 26b, 30b, 32a, 34a, 44
  - **Write down all the steps** that you have done to obtain your answers.
  - Due date: July 9, 2009 (Thursday)
- Today
  - Use handout from lecture 8 first.
  - The new handout is for Chapter 5 (except the first few slides).

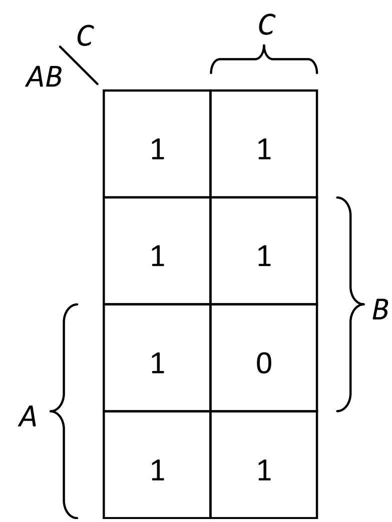
Caution

When you see $\overline{A}\,\overline{B}C$ or $\overline{A}\,\overline{B}\,\overline{C}$ on quiz/HW/exam, please always **double-check** whether the bars on the top are disconnected.

This is the K-map for $X=\overline{A}\,\overline{B}\,\overline{C}$ which is the same as $X=\overline{A}\cdot\overline{B}\cdot\overline{C}$



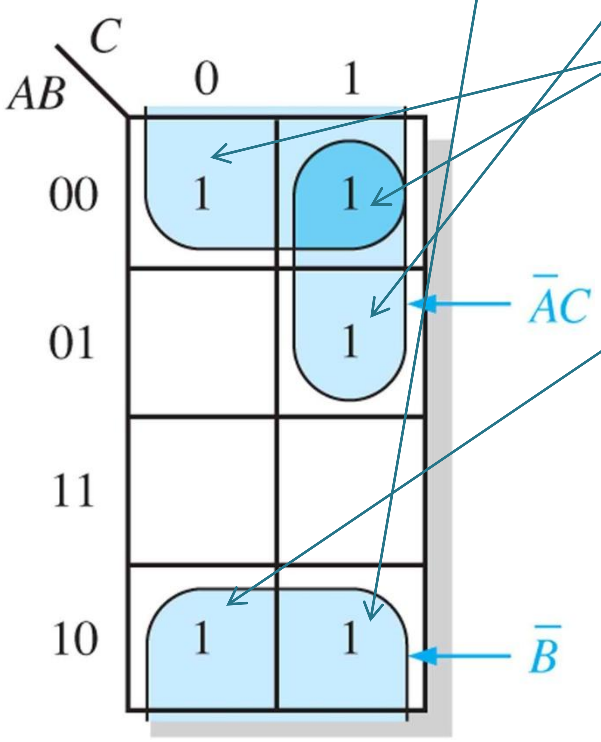
This is the K-map for $X=\overline{ABC}$ which is equivalent to $X=\overline{A}+\overline{B}+\overline{C}$



Example

Use a K-map to minimize the following expression

$$X = A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C}$$

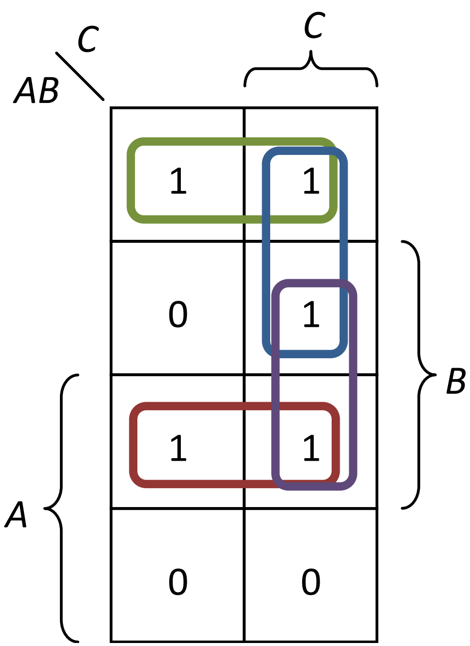


$$X = \overline{B} + AC$$

Non-uniqueness

Use a K-map to minimize the following expression

$$AB + \overline{A}\,\overline{B} + \overline{A}BC$$



Solution 1: $AB + \overline{A}\,\overline{B} + \overline{A}C$

Solution 2: $AB + \overline{A}\,\overline{B} + BC$

"Don't Care" Input Combinations

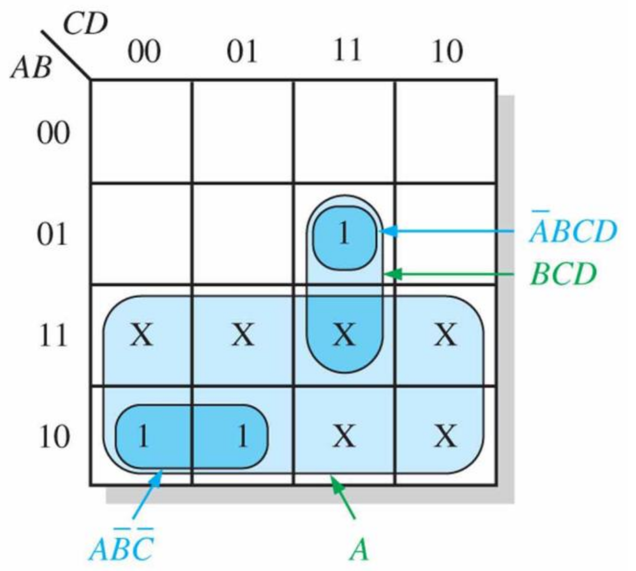
- Sometimes the output doesn't matter for certain input combinations.
  - For example, the combinations are not allowed in the first place.
- These combinations are called "don't care".
- The "don't care" term can be used to advantage on K-map.
- For each "don't care" term, place an X in the corresponding cell.
- When grouping the 1s,
  - the Xs can be treated as 1s to make a larger grouping
  - or as 0s if they cannot be used to advantage.

Example

INPUTS				OUTPUT
A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Don't cares

(a) Truth table



(b) Without "don't cares" $Y = A\bar{B}\bar{C} + \bar{A}BCD$
With "don't cares" $Y = A + BCD$

Alternative Methods

- Disadvantages of using K-maps
  - Not applicable for more than five variables
  - Practical only for up to four variables
  - Difficult to automated in a computer program
- There are other ways to minimize Boolean functions.
  - More practical for more than four variables
  - Easily implemented with a computer
  1. Quine-McClusky method
     - Inefficient in terms of processing time and memory usage
  2. Espresso Algorithm
     - de facto standard

New Perspective: 0

- So far, all of our techniques focus on the 1s in the truth tables/K-maps.
- We can look at the 0s as well.

Caution: From this perspective, you are in a different world. In fact, it is a dual world. Techniques used here will be the dual of what we used before.

Canonical Product

- Product-of-Sums (POS) Form

  Example: $\left(A+\overline{B}\right)\cdot\left(A+B+C\right)$
- Standard POS Form (Canonical Product)

  Example: $\left(A+\overline{B}+C\right)\cdot\left(A+\overline{B}+\overline{C}\right)\cdot\left(A+B+C\right)$
- Convert expression in POS form into canonical product:

Hint:

$$
\begin{aligned}
X &= X+0 \\
&= X+Y\cdot\overline{Y} \\
&= \left(X+Y\right)\cdot\left(X+\overline{Y}\right)
\end{aligned}
$$

Truth Table for Canonical Product

Find the value of X for all possible values of the variables when

$$X = \left(A+\overline{B}+C\right)\cdot\left(\overline{A}+B+C\right)\cdot\left(\overline{A}+\overline{B}+C\right)$$

Old way: Convert to SOP form

$$\begin{aligned}
X &= \left(A+\overline{B}+C\right)\cdot\left(\overline{A}+B+C\right)\cdot\left(\overline{A}+\overline{B}+C\right)\\
&= \left(\left(A+\overline{B}\right)\cdot\left(\overline{A}+B\right)\cdot\left(\overline{A}+\overline{B}\right)\right)+C\\
&= \left(\left(A+\overline{B}\right)\cdot\left(\overline{A}+\left(B\cdot\overline{B}\right)\right)\right)+C\\
&= \left(\left(A+\overline{B}\right)\cdot\overline{A}\right)+C\\
&= \left(\overline{A}\cdot\overline{B}\right)+C
\end{aligned}$$

$\overline{A}\cdot\overline{B}$ $\quad C$

A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Then, construct the truth table.

We can use the property of sum terms to construct the truth table directly.

Maxterm

- A sumterm in a canonical product is called a **maxterm**.
- A maxterm is equal to 0 for only one combination of variable values.

$$A+\overline{B}+C=0 \text{ iff } (A,B,C)=(0,1,0)$$

$$A+\overline{B}+\overline{C}=0 \text{ iff } (A,B,C)=(0,1,1)$$

$$A+B+C=0 \text{ iff } (A,B,C)=(0,0,0)$$

- We say that the maxterm $A+\overline{B}+C$ has a binary value of 010 (decimal 2)
- Maxterm list: $\left(A+\overline{B}\right)\cdot\left(A+B+C\right)=\prod_{A,B,C}(0,2,3)$

because

$$\left(A+\overline{B}\right)\cdot\left(A+B+C\right)=\left(A+\overline{B}+C\right)\cdot\left(A+\overline{B}+\overline{C}\right)\cdot\left(A+B+C\right)$$

Truth Table for Canonical Product

Find the value of X for all possible values of the variables when

$$X = \left(A+\overline{B}+C\right)\cdot\left(\overline{A}+B+C\right)\cdot\left(\overline{A}+\overline{B}+C\right)$$

New way:

A	B	C	X	
0	0	0	1	
0	0	1	1	
0	1	0	0	$A+\overline{B}+C$
0	1	1	1	
1	0	0	0	$\overline{A}+B+C$
1	0	1	1	
1	1	0	0	$\overline{A}+\overline{B}+C$
1	1	1	1	

Minterm/Maxterm & Truth Table

Row #	*A*	*B*	*C*	*Minterm*	*Maxterm*
0	0	0	0	$\overline{A}\cdot\overline{B}\cdot\overline{C}$	$A+B+C$
1	0	0	1	$\overline{A}\cdot\overline{B}\cdot C$	$A+B+\overline{C}$
2	0	1	0	$\overline{A}\cdot B\cdot\overline{C}$	$A+\overline{B}+C$
3	0	1	1	$\overline{A}\cdot B\cdot C$	$A+\overline{B}+\overline{C}$
4	1	0	0	$A\cdot\overline{B}\cdot\overline{C}$	$\overline{A}+B+C$
5	1	0	1	$A\cdot\overline{B}\cdot C$	$\overline{A}+B+\overline{C}$
6	1	1	0	$A\cdot B\cdot\overline{C}$	$\overline{A}+\overline{B}+C$
7	1	1	1	$A\cdot B\cdot C$	$\overline{A}+\overline{B}+\overline{C}$

"1" "0"

In the same way that each minterm corresponds to a unique row of the truth table,

each maxterm corresponds to a unique row of the truth table (in a dual way).

Conversion

Row #	*A*	*B*	*C*	*Minterm*	*Maxterm*
0	0	0	0	$\overline{A}\cdot\overline{B}\cdot\overline{C}$	$A+B+C$
1	0	0	1	$\overline{A}\cdot\overline{B}\cdot C$	$A+B+\overline{C}$
2	0	1	0	$\overline{A}\cdot B\cdot\overline{C}$	$A+\overline{B}+C$
3	0	1	1	$\overline{A}\cdot B\cdot C$	$A+\overline{B}+\overline{C}$
4	1	0	0	$A\cdot\overline{B}\cdot\overline{C}$	$\overline{A}+B+C$
5	1	0	1	$A\cdot\overline{B}\cdot C$	$\overline{A}+B+\overline{C}$
6	1	1	0	$A\cdot B\cdot\overline{C}$	$\overline{A}+\overline{B}+C$
7	1	1	1	$A\cdot B\cdot C$	$\overline{A}+\overline{B}+\overline{C}$

This tells that the output column of the truth table is 1 on row # 0, 1, 2, 3.

This tells that the output column of the truth table is 0 on row # 4, 5, 6, 7.

$$\Sigma_{A,B,C}(0,1,2,3) = \Pi_{A,B,C}(4,5,6,7)$$

$$\Sigma_{X,Y}(1) = \Pi_{X,Y}(0,2,3)$$

$$\Sigma_{W,X,Y,Z}(0,1,2,3,5,7,11,13) = \Pi_{W,X,Y,Z}(4,6,8,9,10,12,14,15)$$

K-Map POS Minimization

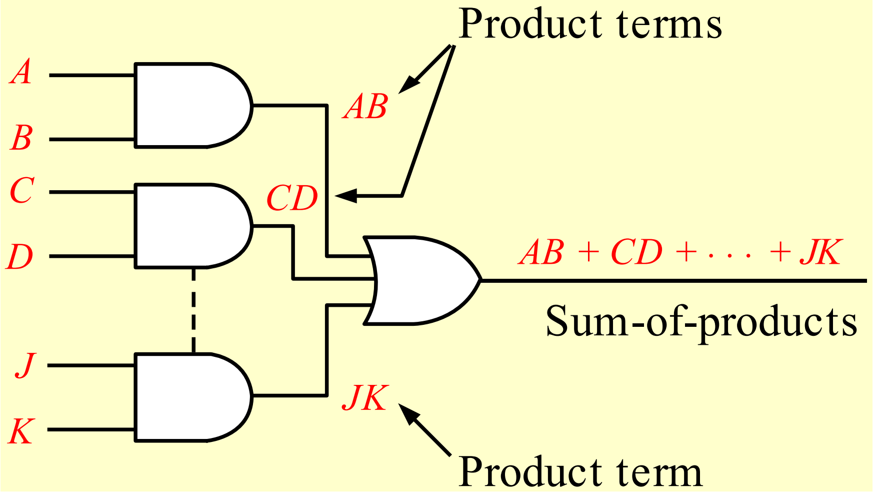
- Goal: Find the “Minimal Product”
- Appendix B in the textbook.
- For a POS expression in standard form, a 0 is placed on the K-map for each sumterm in the expression.
- The cells that do not have a 0 are the cells for which the expression is 1.
- Group 0s to produce instead of grouping 1s.

Combinational Logic

- Chapter 5 and 6
- Reading Assignment:
  - Read Section 5-1 to 5-5.
- Definition: A **combinational logic** is a combination of logic gates interconnected to produce a specified Boolean function with no storage or memory capability.
- Sometimes called **combinatorial logic**.

SOP Implementation: AND-OR Circuit

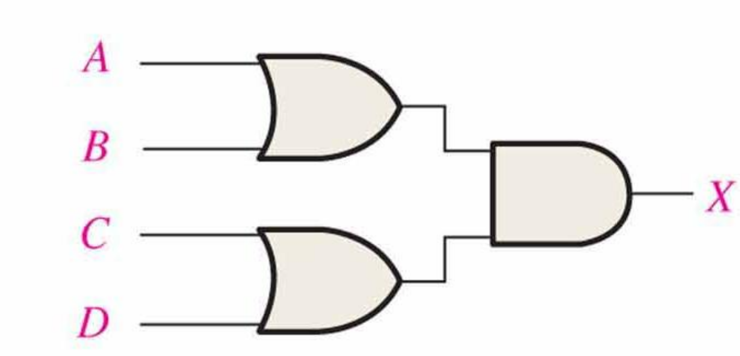
In Sum-of-Products (SOP) form, basic combinational circuits can be directly implemented with AND-OR combinations: first forming the AND terms; then the terms are ORed together.



This is called the **AND-OR configuration**.

Example

Write the output expression of the following circuit as it appears in the figure and then change it to an equivalent AND-OR configuration.

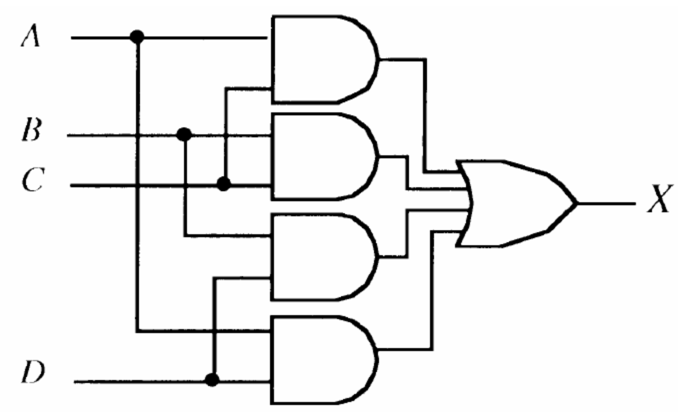


Solution:

$$X = (A+B)\cdot(C+D)$$

$$= (A+B)\cdot C + (A+B)\cdot D$$

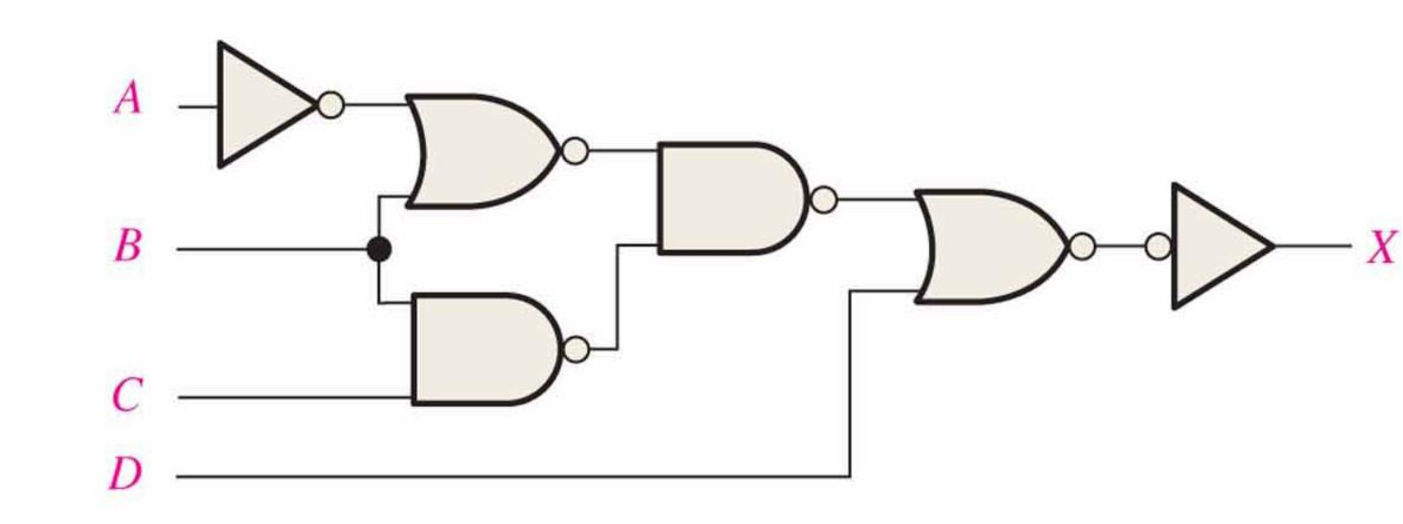
$$= AC + BC + AD + BD$$



(a)

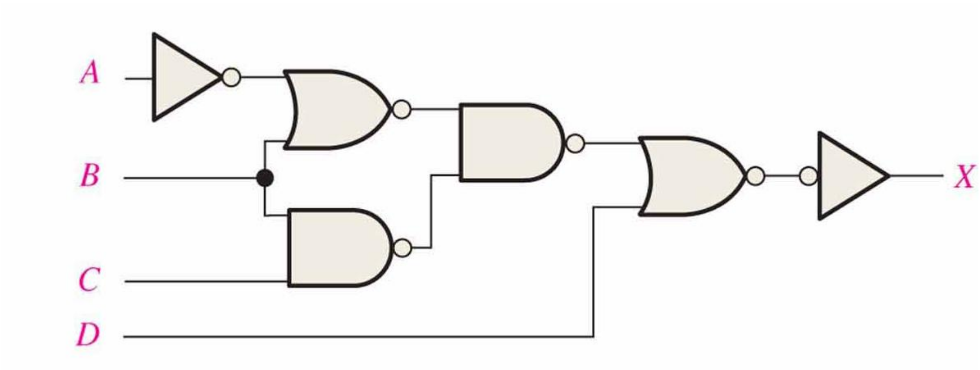
Example

Write the output expression of the following circuit as it appears in the figure and then change it to an equivalent AND-OR configuration.



Solution

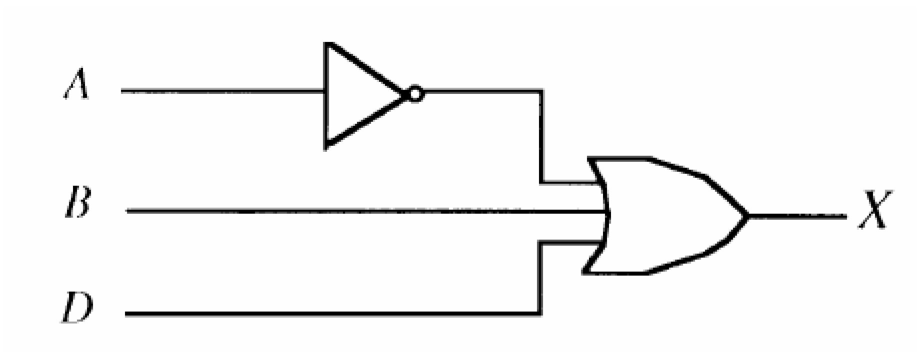
$$X = \overline{\overline{\overline{\left(\overline{\overline{A}+B}\right)\cdot\left(\overline{B\cdot C}\right)}+D}}$$



$$= \overline{\left(\overline{\overline{A}+B}\right)\cdot\left(\overline{B\cdot C}\right)}+D$$

$$= \overline{A}+B+B\cdot C+D$$

$$= \overline{A}+B+D$$



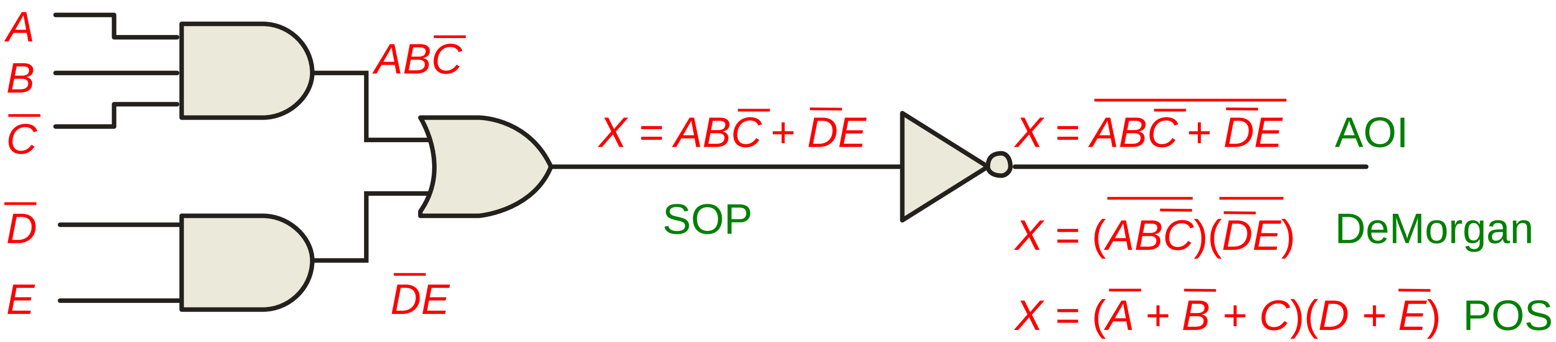
Remark

1. From any logic expression, you can construct a truth table.
2. From the truth table you can get a canonical sum or a minterm list. (This can be simplified to a minimal sum. In any case, you get a SOP expression)
3. Any SOP expression can be implemented using AND gates, OR gates, and inverters.

AND-OR-Invert (AOI) circuit

When the output of a SOP form is inverted, the circuit is called an **AND-OR-Invert circuit**.

The AOI configuration lends itself to product-of-sums (POS) implementation.

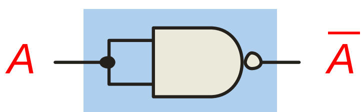


Universal gate

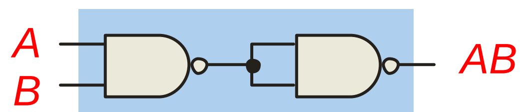
- The term **universal** refers to a property of a gate that permits any logic function to be implemented by that gate or by a combination of gates of that kind.
- Example: NAND gates, NOR gates

NAND Gate as a Universal Gate

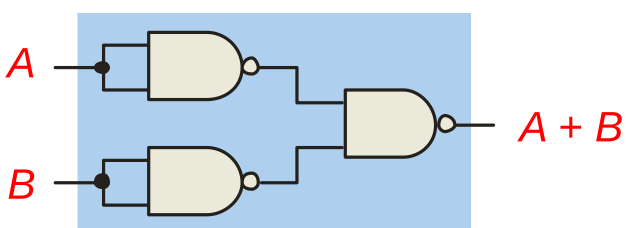
NAND gates are sometimes called **universal** gates because they can be used to produce the other basic Boolean functions.



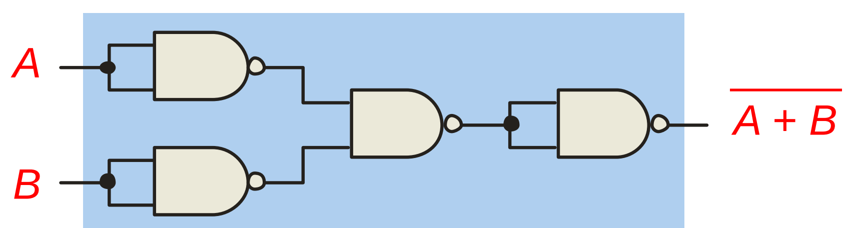
Inverter



AND gate



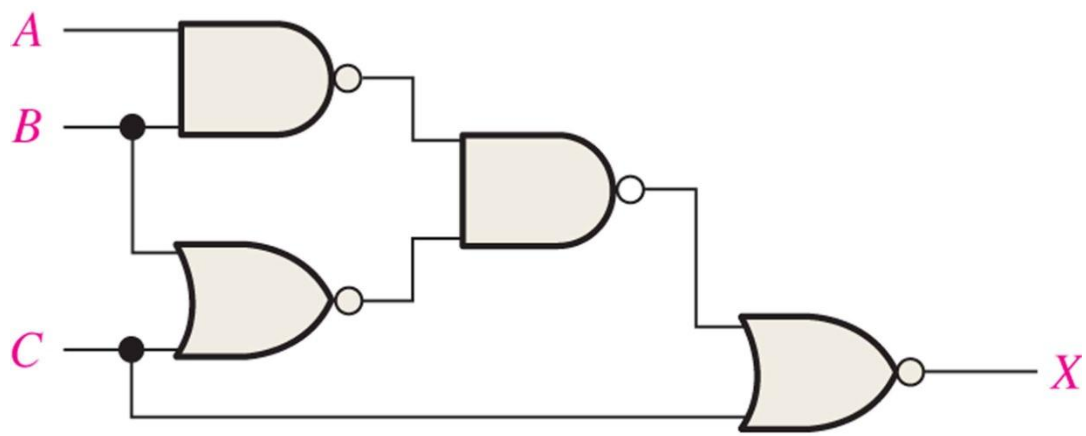
OR gate



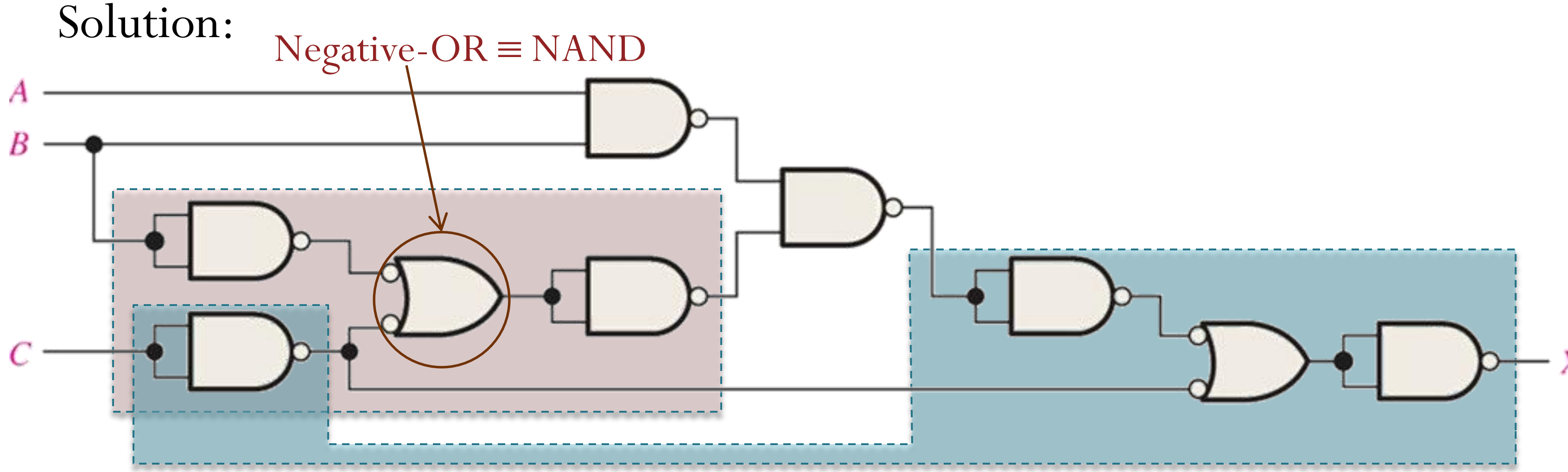
NOR gate

Example

Implement the following logic circuit using only NAND gates:

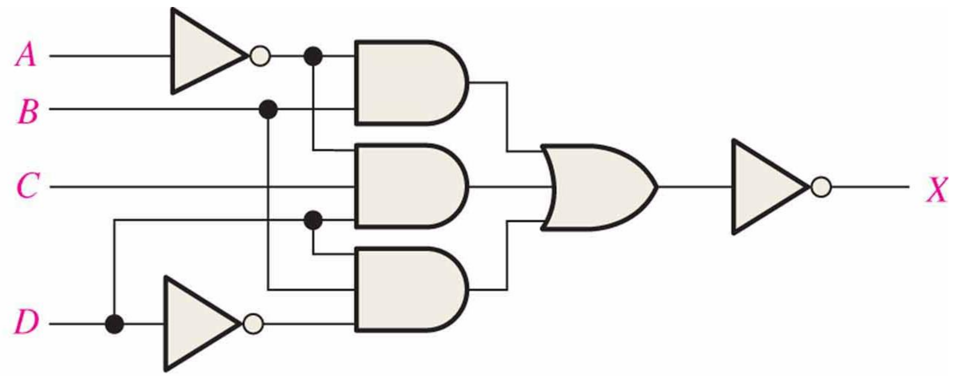


Solution:

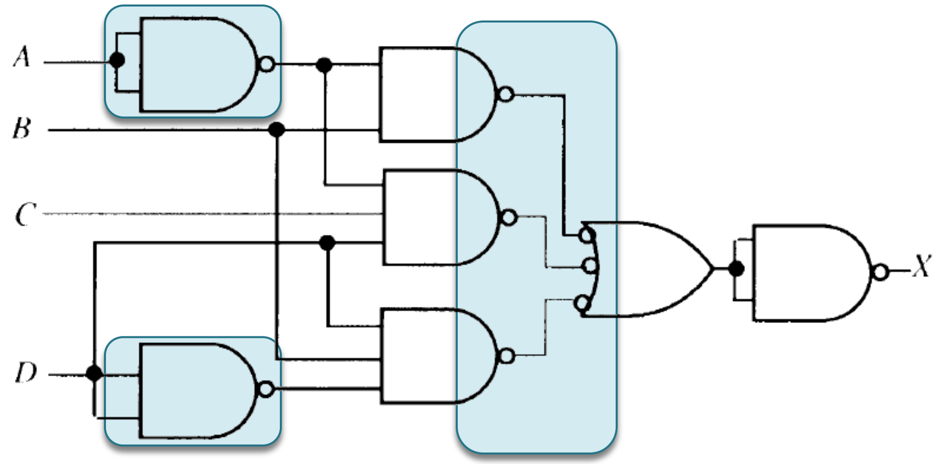


Example

Implement the following logic circuit using only NAND gates:

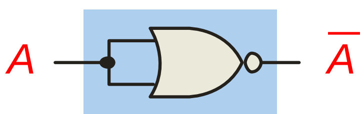


Solution:

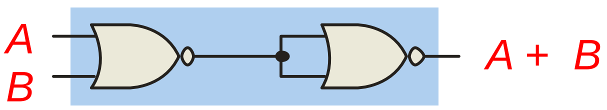


NOR Gate as a Universal Gate

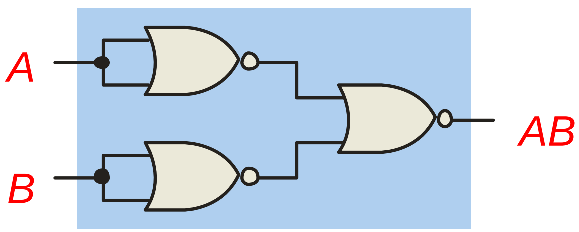
NOR gates are also **universal** gates and can form all of the basic gates.



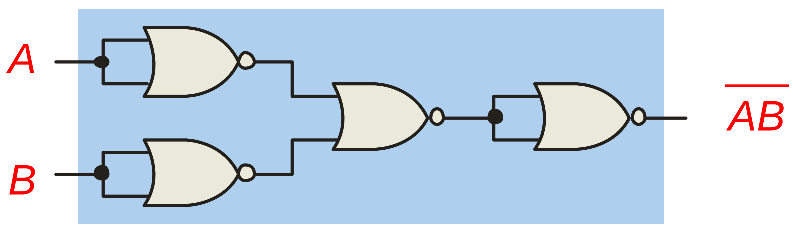
Inverter



OR gate



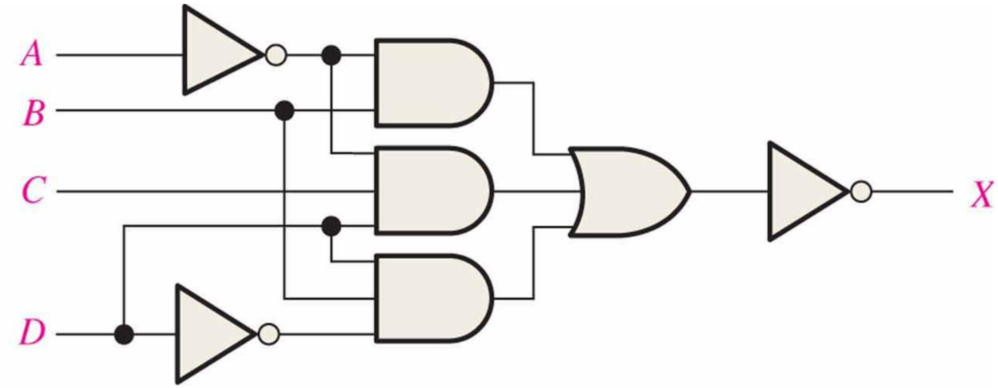
AND gate



NAND gate

Example

Implement the following logic circuit using only NOR gates:



Solution: